Enrollment No: _____ Exam Seat No:_____

C.U.SHAH UNIVERSITY

Summer Examination-2018

Subject Name : Engineering Mathematics - II

Subject Code : 4TE02EMT2 **Branch: B.Tech (All)**

Semester : 2 **Date : 25/04/2018** **Time : 10:30 To 01:30** **Marks : 70**

Instructions:

(1) Use of Programmable calculator & any other electronic instrument is prohibited.
(2) Instructions written on main answer book are strictly to be obeyed.
(3) Draw neat diagrams and figures (if necessary) at right places.
(4) Assume suitable data if needed.

Q-1 **Attempt the following questions:** **(14)**

a) The value of $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} \cos^7 x\, dx$ is

(A) $\frac{32\pi}{35}$ (B) $\frac{32}{35}$ (C) zero (D) $\frac{16}{35}$

b) If $f_n = \int_0^{\frac{\pi}{4}} \tan^n x\, dx$, then $\left(f_n + f_{n-2}\right)$ is equal to _____.

(A) $\frac{1}{n}$ (B) $\frac{1}{n-1}$ (C) $\frac{n}{n-1}$ (D) $\frac{n-1}{n}$

c) $B(1,1) =$ _____

(A) 1 (B) 0 (C) 1/2 (D) none of these

d) $\left\lfloor\frac{1}{4}\right\rfloor\left\lfloor\frac{3}{4}\right\rfloor =$ _____

(A) $\frac{\pi}{\sqrt{2}}$ (B) $\pi\sqrt{2}$ (C) $\sqrt{2\pi}$ (D) none of these

e) $\int_0^{\frac{\pi}{2}} \frac{d\theta}{\sqrt{1-2\sin^2\theta}}$ is equal to

(A) $\frac{1}{\sqrt{2}}E\left(\frac{1}{\sqrt{2}}\right)$ (B) $\frac{1}{2}K\left(\frac{1}{\sqrt{2}}\right)$ (C) $\frac{1}{\sqrt{2}}K\left(\frac{1}{\sqrt{2}}\right)$ (D) $\frac{1}{2}E\left(\frac{1}{\sqrt{2}}\right)$

f) $erf(-x)$ is

(A) an odd function (B) an even function (C) neither odd nor even function
(D) none of these

g) If the power of x are even, then the curve is symmetrical about

(A) X-axis (B) Y-axis (C) about both X and Y axes (D) none of these

h) The curve $x^3+y^3=3axy$ represent

(A) Cissoid of Diocle (B) Witch of Agnesi (C) Strophoid

(D) Folium of Descartes

i) $\int_0^a \int_0^{\sqrt{a^2-y^2}} dx\,dy$ is equal to

(A) πa^2 (B) $\frac{\pi a^2}{2}$ (C) $\frac{\pi a^2}{4}$ (D) none of these

j) On converting into polar coordinates $\int_0^{2a} \int_0^{\sqrt{2ax-x^2}} dx\,dy$ is equal to

(A) $\int_0^{\pi} \int_0^{2a\cos\theta} r\,dr\,d\theta$ (B) $\int_0^{\frac{\pi}{2}} \int_0^{2a\cos\theta} r\,dr\,d\theta$ (C) $\int_0^{\frac{\pi}{2}} \int_0^{2a\sin\theta} r\,dr\,d\theta$ (D) none of these

k) The degree and order of the differential equation of all parabolas whose axis is x-axis are

(A) 2, 1 (B) 1, 2 (C) 3, 2 (D) none of these

l) The solution of the equation $(x+y)^2\frac{dy}{dx}=a^2$ is

(A) $(x+y)-a\tan^{-1}\left(\frac{x+y}{a}\right)=x+c$ (B) $(x+y)a\tan x=x+c$

(C) $(x+y)-a\cos a=x+c$ (D) none of these

m) The infinite series $1+r+r^2+.....+r^{n-1}+....$ is divergent if

(A) $|r|<1$ (B) $|r|>1$ (C) $r\geq 1$ (D) $r=-1$

n) The sum of the series $\sum_{n=1}^{\infty}(-1)^{n-1}\frac{1}{n}=1-\frac{1}{2}+\frac{1}{3}-\frac{1}{4}+.....$ is

(A) $\log 2$ (B) zero (C) infinite (D) none of these

Attempt any four questions from Q-2 to Q-8

Q-2 **Attempt all questions** **(14)**

a) Using reduction formula prove that $\int_0^{\pi} x\sin^7 x\cos^4 x\,dx=\frac{16\pi}{1155}$ **(5)**

b) Evaluate: $\int_0^{\infty} x^4e^{-x^4}dx$ **(5)**

c) Evaluate: $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dx\,dy\,dz}{\sqrt{1-x^2-y^2-z^2}}$ **(4)**

Q-3 **Attempt all questions** **(14)**

a) Show that $\int_0^1 \frac{dx}{\sqrt{1-x^n}}=\frac{\sqrt{\pi}}{n}\cdot\frac{\left\lfloor\frac{1}{n}\right\rfloor}{\left\lfloor\frac{1}{n}+\frac{1}{2}\right\rfloor}$ **(5)**

b) Solve: $xdy-ydx=\sqrt{x^2+y^2}dx$ **(5)**

c) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{\tan^{-1} n}{n^2+1}$. **(4)**

Q-4 **Attempt all questions** **(14)**

a) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} \left(x^2+y^2\right) dx\, dy$ by changing into polar coordinates. **(5)**

b) Test for convergence the series $\sum_{n=1}^{\infty} \frac{\left[(n+1)x\right]^n}{n^{n+1}}$. **(5)**

c) Evaluate: $\int_0^{\frac{\pi}{6}} \cos^6 3\theta \sin^2 6\theta \, d\theta$ **(4)**

Q-5 **Attempt all questions** **(14)**

a) Solve: $\frac{dy}{dx} + 2y\tan x = \sin x$ given that $y = 0$ when $x = \frac{\pi}{3}$. **(5)**

b) Change the order of integration in the integral $\int_0^{\infty}\int_x^{\infty} \frac{e^{-y}}{y} dy\, dx$ and evaluate it. **(5)**

c) Prove that $B(m,n) = B(m,n+1) + B(m+1,n)$ **(4)**

Q-6 **Attempt all questions** **(14)**

a) Test the convergence of the series $\frac{1}{1\cdot 2\cdot 3} + \frac{3}{2\cdot 3\cdot 4} + \frac{5}{3\cdot 4\cdot 5} + ...$ **(5)**

b) Using reduction formula prove that $\int_0^a x^5 \left(2a^2 - x^2\right)^{-3} dx = \frac{1}{2}\left(\log 2 - \frac{1}{2}\right)$. **(5)**

c) Solve: $\left(y^2 e^{xy^2} + 4x^3\right)dx + \left(2xye^{xy^2} - 3y^2\right)dy = 0$ **(4)**

Q-7 **Attempt all questions** **(14)**

a) Trace the curve $y^2(2a - x) = x^3$. **(5)**

b) Find the volume of the solid generated by the revolution of the loop of the curve $x\left(x^2+y^2\right) = a\left(x^2-y^2\right)$ about the x – axis. **(5)**

c) Evaluate: $\int_0^{\frac{\pi}{2}} \frac{dx}{\sqrt{\cos x}}$ **(4)**

Q-8 **Attempt all questions** **(14)**

a) For small values of x, show that $\operatorname{erf}(x) = \frac{2}{\sqrt{\pi}}\left(x - \frac{x^3}{1!3} + \frac{x^5}{2!5} - \frac{x^7}{3!7} + ...\right)$. **(5)**

b) Trace the curve $r^2 = a^2 \cos 2\theta$. **(5)**

c) Find the length of the arc of the curve $y = \log\sec x$ from $x = 0$ to $x = \frac{\pi}{3}$. **(4)**